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**To:** Finance Committee, Chilean Senate  
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## Creating an Auction for Chilean Fisheries

### Executive Summary

Well-designed licenses and auction rules have been successfully used worldwide to improve the allocations of public resources, including radio spectrum licenses, mining rights and fisheries licenses.<sup>1</sup> But Chile's current auction for Class B fisheries licenses – and the auction rules proposed in the Ley Corta – make bidding unnecessarily difficult, especially for smaller and new participants, and fail to maximize competition and revenue. These auctions are likely to produce an inefficient allocation of public resources, thus failing to maximize the welfare of citizens of Chile.

To ensure that licenses match bidder needs and make participation as easy as possible, Chile should define each license to combine each species with its accompanying fauna. The units licensed should be small – for example, one million units could correspond to one percent of the total available catch. Chile should allocate all licenses simultaneously, in a single auction sale, and should adopt an auction design that allows bidders to express their preferences for different combinations of licenses, including substitution possibilities among licenses.

These requirements can be fulfilled by using the rules of a *sealed bid, uniform price assignment auction*.<sup>2</sup> By design, this auction makes it easy for bidders to bid simultaneously for different alternative licenses according to their preferences and allows bidders to vary demand across licenses according to which is most favorably priced by the auction. The design spares bidders from having to guess which licenses will be relatively less expensive, ensures that all bidders pay the same prices for equal licenses, and guarantees that bidders whose bids match their values always win their most preferred licenses given the final prices. These improvements to the license definition and auction design would substantially improve the competitiveness, fairness, simplicity and realized revenues of the allocation of Chilean fisheries licenses.

An appendix to this report includes language that might be used in the law to describe the proposed auction rules.

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<sup>1</sup> Successful rights auctions involving fisheries include the New Zealand auctions and geoduck auctions in Washington, USA.

<sup>2</sup> The mathematical formulation and analysis of the assignment auction is described in an appendix and is based on my paper "Assignment Messages and Exchanges," *American Economic Journal: Microeconomics*, Vol 1, Issue 2, 2009, pages 95–113.

## 1. Introduction and Goals

This report proposes a cohesive set of rules for an auction to allocate licenses for Chilean fisheries.

The proposed auction design can achieve important goals that support an improved allocation of fishing quotas. If implemented correctly and fully, this auction design can:

1. Capture a significant portion of the value of the fisheries resource for citizens of Chile.
2. Enable all bidders to compete fairly, paying equal prices for the same fishing quotas.
3. Make bidding easy, to promote competition and entry and to allow efficient quota allocations.
4. Limit concentration in the fishing industry and its damaging effects on related sectors.

## 2. Context and Precedents

In 2011, I provided a report and presentation that described the key elements needed for a successful auction for fishing quotas and offered auction rules and software that could be applied to the Chilean context.

That recommendation was informed by the historical successes and failures of auctions in related contexts. One important precedent is the auction of radio spectrum licenses. Spectrum licenses share important characteristics with licenses for fishing quotas, which makes the auction design problems similar. Both kinds of licenses grant private actors protected access to a public resource, and both are geographically defined. Both radio spectrum licenses and fishing quotas can involve multiple licenses in each area (frequencies or fish species), and both may involve licenses that are partly interchangeable: users are willing to trade among licenses that provide alternative ways to serve their needs.

Auctions for radio spectrum licenses, using designs that my collaborators and I created and refined in the 1990s and 2000s, have been widely hailed as successful and have become common worldwide. Well-designed spectrum auctions produce efficient allocations and high revenues for governments. Following those examples, auctions have been successfully used to allocate other resources, including commodities, electricity, mining, and internet resources. My work on auction design was recognized by the award of a Nobel Prize in Economic Sciences in 2020.

There have also been notable auction failures. In most cases, failures can be attributed to poor auction design that promoted corruption, collusion, or difficult or manipulative bidding. In designing auction rules and the licenses to be sold, it is important to avoid a 'one size fits all' approach – instead, the details of a particular resource context are very important. In 2011, I proposed that a sealed bid, uniform price assignment auction with properly designed licenses could succeed within the specific context of Chile's fisheries.

The design and recommendations that I offered in 2011 were not adopted. Although Chile has since introduced limited auctions for Class B licenses, the design of that auction – and the definition and overall supply of the licenses that it allocates – make participation difficult for bidders and do not promote an efficient allocation. Many problems in the broader market persist, including the concentration of licenses among a small number of incumbents.

### 3. Problems with Current and Proposed Auction Rules

#### Current Auction for Class B Licenses

The current auction design for Class B licenses is insufficient to meet the regulatory and efficiency goals for Chilean fisheries. The standard of comparison is simple: an appropriate fisheries auction should [1] minimize bidders' need to guess about the demands of others, making it safe for a bidder to bid truthfully for the licenses that it hopes to buy, [2] allow a bidder to win larger quantities of licenses when prices are lower, [3] allow a bidder to win licenses for species and regions that it finds relatively more attractive at the auction-determined prices, and generally [4] allocate to each bidder its most preferred set of licenses at the auction-determined prices. The current Class B license auction design does none of these things, but the auction recommended in this report does them all. As discussed below, it does more, as well.

Here are some of the main failures of the current auction for Class B licenses.

*First*, the current auction **does not allow bidders to simultaneously purchase a license and the required accompanying fauna license**. The value of a license for one species alone likely depends on the price and availability of the accompanying species license. Under the current design, straightforward bidding is unsafe: a bidder might acquire a license for jack mackerel but be unable to use it, because it does not acquire the accompanying fauna licenses for anchovy, common sardine, and hoki. This risk makes licenses for each species less valuable to all bidders, and especially new entrants, leading them to reduce their bids and resulting in less effective competition, lower auction revenues and diminished efficiency.

*Second*, the current auction for Class B licenses **does not allow bidders to demand different quantities of a license at different prices**. It is not possible to express the demand of a bidder who wishes to say, "I want to acquire more licenses for sardines in regions V–X if the auction-determined price of these licenses is sufficiently low." This failure makes bidding unnecessarily difficult, especially for small bidders and new entrants, and makes it impossible to submit bids that fully express a bidder's actual preferences. Because a bidder cannot adjust its quantity demand based on the auction-determined license price, it may acquire too much or too little at the final price. Its bids must be based on guesses, which introduce unnecessary randomness and lead to an inefficient allocation.

*Third*, the current auction for Class B licenses **does not allow bidders to substitute among different types of quotas**, depending on the prices of those quotas. A bidder might wish to say, "I want a license for jack mackerel either in regions III–IV or in regions V–IX, according to which of the two is the better value for me." Under the present design, a bidder must bid for jack mackerel in regions III–IV and regions V–IX independently, which makes bidding extremely risky. The bidder must guess which will be the better deal and, if it bids for both, it risks winning too much and paying more than it can afford. The typical way that bidders respond to this risk is to reduce their bids to avoid spending too much. That leads to less competition, lower auction revenues and an inefficient allocation.

*Fourth*, the current auction for Class B licenses **creates entry barriers and fails to limit concentration in the industry**. There is no set-aside or overall quota accumulation limit. Bidding deadlines make it difficult to prepare for and participate in the auction, especially for small bidders and new entrants. Lots are not small enough to allow different bidders to purchase their desired quantities. These features are likely to increase concentration and reduce competition in the fisheries market, which in turn harms Chileans that rely on fisheries resources.

These failures of the current auction for Class B licenses create unnecessary complications for bidders and the risk of an unfair and inefficient allocation of fisheries licenses, but they can be eliminated by a more appropriate auction design.

### Proposed Ley Corta

The proposed law does not correct the problems with the current auction for Class B licenses and, in addition, would inefficiently and unnecessarily use a sequence of sales of identical licenses over a period of years. This plan discourages an efficient allocation by making it harder for a bidder seeking to achieve efficient scale to acquire a sufficient set of licenses and requiring bidders in earlier auctions to guess about future license prices (which makes bidding harder and harmful errors more likely). While the law sets broad goals that are consistent with good auction design, it is lacking in many important details and needs critical improvements to the product design, auction timing, competitive measures, and bidding rules.

## 4. An Improved Auction

Each of the problems with the current auction for class B licenses and the proposed Ley Corta can be addressed by properly defining the licenses and adopting an appropriate auction design.

### Five Conditions

The law should require that the final auction design satisfies the following five conditions. Well-conceived auction rules – which govern how bids are expressed and how winners and prices are determined – are necessary but not sufficient for a good auction design. Of equal importance are other preconditions: the overall supply of the resource; the detailed license definition; the broader rules for participation; and pro-competitive market measures.

*First, the products to be sold should be defined in a way that **accommodates bidder preferences and makes the licenses usable.***

- A. Products should be defined by three characteristics: [1] the mix of species (primary species and accompanying species), [2] the region, and [3] the category of eligible bidders. All licenses should be identical and sufficiently small to allow all bidders to compete for the same licenses.<sup>3</sup>
- B. All licenses should incorporate accompanying fauna directly, to make the licenses usable. Failing to package accompanying fauna makes licenses less usable, bidding riskier, and entry more difficult.
- C. All licenses except set-aside licenses should be fully tradeable.

*Second, the auction should **allocate, in a single sale, the full quota of available fishing licenses.***

The licenses should not be sold in multiple auctions, sequentially or fractionally. Splitting the quota among auctions over time results in different bidders paying different prices for the same license, which tends to result in less efficient, less fair allocations. It makes bidding harder by forcing bidders to guess about future prices when bidding in the current auction, creates risks for entry, and makes it easier for incumbents to blockade entry. Because these features tend to discourage participation and induce bidders to bid more cautiously, multiple auctions are also likely to result in lower revenue.

*Third, bidding rules should **allow bidders to make their demands for the licenses on sale responsive to the auction-determined prices**, including demands that may substitute among different licenses that may meet their needs.*

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<sup>3</sup> This same license design principle was applied successfully for auctions of radio spectrum licenses in the USA, UK, and Canada earlier in 2021.

*Fourth*, there should be a **reasonable, workable timetable for auction preparations**. Bidders need sufficient time to understand and practice with the auction rules, establish and refine their preferences, and prepare their bids. Failing to announce rules and procedures long enough in advance and to allow sufficient preparation time reduces participation and entry, which can in turn restrict the efficiency of the allocation and the revenue raised.

*Fifth*, the auction should define and **distinguish large incumbents, and establish a set aside for smaller bidders and/or entrants**. This set aside would reserve a percentage of the available supply – perhaps 25%<sup>4</sup> – for smaller qualified bidders, ensuring that they can establish operations even if there is substantial competition among the large bidders. Failing to set aside quotas for small bidders and new entrant enables blocking strategies by large incumbents, increases market concentration and discourages participation.

### The Assignment Auction

To make bidding both easy and flexible, and to promote an efficient allocation, Chile should adopt a *sealed bid, uniform price assignment auction*.

In a *sealed-bid* auction, each bidder submits a collection of bids that, taken together with the bids of others, determines the licenses that each bidder wins and the prices that will be charged. A sealed-bid design – as compared to multi-round auctions that have been used in some settings – is familiar in Chile and other fisheries, makes participation and bidding simpler for bidders, makes implementation easier for the regulator, and allows the auction to be completed much faster (in hours instead of weeks or months). Moreover, compared to a multi-round auction, a sealed-bid auction makes collusion among bidders and entry-detering strategies harder, because a bidder cannot observe or infer the behavior of competitors and react to its observations in later bidding rounds.

The *assignment auction* is a flexible auction design that allows each bidder to make the products and quantities it demands vary with market prices when the goods offered for sale are substitutable. It accomplishes this using base bids, bid groups, weights and limits, as described below.

- Each **base bid** is a product-price-quantity triple. For example, a bid of (product X, 5, \$200) says that “if the price is no more than \$200, I want 5 units of product X.” No bidder is required to submit bids of any other kind, so bidding in this auction need not be more difficult than it is under the existing rules for Class B licenses.
- Each bidder may, at its option, submit **multiple** base bids for each product, which allows its demand for the product to vary with the price of the product. For example, if in addition to the bid of (product X, 5, \$200) described above, the bidder also submits a lower-priced bid of (product X, 2, \$150), that means that “if the price of X is less than \$150, then I want an additional 2 units of product X.” Taken together, the two bids indicate that the bidder demands 7 units of product X if the price is lower than \$150; 5 units of product X if the price is higher than \$150 and lower than \$200; and no units of product X if the price is higher than \$200. Of course, a bidder can express an even richer demand for product X by submitting additional base bids.
- Each bidder also has the option to specify substitution among its bids for different products. This is accomplished by:
  - Placing multiple base bids for different products in a single **bid group**;
  - Assigning a **weight** to each base bid in a bid group; and

<sup>4</sup> The optimal set-aside percentage depends on a study of the industry to assess what is required and proper to protect smaller bidders and encourage new entry.

- Choosing a group **limit** for the bid group, which restricts the total weighted quantity that the bidder is willing to acquire from all the base bids included in the bid group.

By using these additional features of the assignment auction, a bidder that faces constraints in the total quantity of licenses that it wishes to acquire can safely bid for all products that it is interested in, without facing the risk of winning and buying more than it wants.

For example, suppose that in addition to the two bids described above, the bidder makes an additional base bid for a different product of (product Y, 7, \$200). The bidder might wish to place all those bids in a single bid group and limit the total licenses that it may acquire from this group according to the estimated tonnage of its bids or according to some other criterion. To do that, the bidder can specify: “I apply a weight of 2 to product X and a weight of 3 to product Y and I do not want more than 25 weighted units in total from this set of three bids. Subject to that constraint, I want the auction to assign me the most profitable combination of products and quantities.”

The bids in the examples above would be entered in the bidder’s table of bids as follows:

Product	Max Quantity	Max Price	Weight	Weighted Units
X	5 units	\$200	2	10 w-units
X	2 units	\$150	2	4 w-units
Y	7 units	\$200	3	21 w-units
Group Limit				25 w-units

The **weights** for the various products are useful to express the bidder’s rate of substitution among units of two different products. For example, if the bidder seeks to acquire licenses for a particular tonnage of biomass, the weights could reflect the relative biomass associated with the units of each license. In the example tabulated above, license Y might be estimated to have 50% more associated biomass tonnage than license X, and the group limit of 25 could then reflect the maximum total tonnage that the bidder is willing to buy.

The base bids, weights, and limits combine to determine the bidder’s demand and final allocation at the auction-determined product prices. In the tabulated example, if the auction determines the prices of products X and Y to be  $p_x = \$120$  and  $p_y = \$140$ , respectively, then the auction software will calculate the bidder’s profit on product X to be \$80 per unit for the first five units and \$30 per unit for its sixth and seventh units, and the bidder’s profit on product Y to be \$60 per unit for up to seven units. Given these profits, the bidder demands five units of product X (equivalent to 10 weighted units) at price \$120 and five units of product Y (equivalent to 15 weighted units) at price \$140, respectively. It does not demand additional units of product X, even though those have positive profits, because the demanded units exhaust the bidder’s total limit of 25 weighted units, with a profit of \$700, which is the highest total profit that this bidder can obtain at the auction prices given the limits it has specified. According to the auction rules, the bidder’s allocation is always equal to its demand at the auction-determined prices, so the auction allocation maximizes the bidder’s profit given its bids and limits.

I have just described the auction from a bidder’s perspective. The mechanics of the computation of prices and allocations is described in the attached appendix.

The structure of bids in this auction is more complex than in the existing Chilean rules for Class B licenses. Experience suggests that despite this additional complexity, the net effect of including these options is to make bidding much easier. Bidders who do not wish to use additional features of the assignment auction – the weights, groups, and limits – can choose to bid exactly as they would under the existing rules, specifying a single price and quantity for each product for which they are eligible. Once the rules have been explained, however, most bidders are likely to use the additional options,



because they provide extra flexibility to fully express their demands and eliminate much of the guesswork about which products and quantities to demand and at what prices. Assignment bidding allows a bidder to bid for every combination that it deems relevant to avoid the regret, after the auction, of having bid for the wrong amounts or the wrong product.

In a *uniform price* auction, all bidders pay the same prices for the same products, which promotes a fair and efficient allocation. The assignment auction proposed here solves an optimization problem (a linear program) to compute the minimum prices that are required to clear the market – that is, to ensure that supply equals demand for every product. It allocates to each bidder the mix of products that maximizes that bidder's net value (its bid minus the price it pays) for each product.<sup>5</sup> Under this pricing rule, truthful bidding is a viable strategy: a bidder that bids its true values for each product will acquire its most profitable quantities of each product given the auction-determined prices.

This uniform price auction promotes participation by small bidders and new entrants and encourages them to bid their full values for each product, because they do not face the risk of paying higher prices than better-informed large incumbents. It also encourages them to bid for all the products that may be of interest and to truthfully express their demand across products, because a bidder's bids for different products are prioritized by the auction in order of their profitability for the bidder and limited in total quantity as the bidder instructs.

### Consistent and Coherent Design

Finally, a well-designed auction should incorporate **all** of the first four conditions described above and must use a comprehensive set of auction rules. Failing to adopt any of these conditions – for example, by splitting the allocation over many auctions or failing to bundle licenses with their accompanying fauna – may lead to poor outcomes including an inefficient allocation even with otherwise good auction rules. Similarly, failing to implement good auctions rules – for example, by preventing bidders from specifying maximum amounts to guide substitution across different products – may lead to an inefficient allocation. Failing to allow substitution will force bidders to guess which products are likely to offer the best deals and to withhold other bids, for fear of overspending. Those who guess wrong will be penalized in the auction, which fails to promote efficiency. Also, the missing bids will result in lower prices. In sum, the product definition and auction rules must be well matched to one another to have a successful auction.

The fifth condition described above is a matter of industrial policy, seeking to limit overall concentration in the industry. The auction would establish separate set-aside products for which only eligible bidders could submit bids. The remaining “unrestricted” products could be allocated to any bidder, including a set-aside bidder.

## 5. Conclusion

Auctions of public resources around the world have largely succeeded or failed according to whether the design satisfies the needs of the application. For Chilean fisheries, bidders need to be able to bid on usable licenses in sufficient quantities, to substitute among species and areas according to the prices of the various products, and to have ample time and certainty to prepare and bid. The design proposed in this report meets the needs of this application and can promote entry and efficient outcomes and capture a significant portion of the value of the public fisheries resource for the people of Chile.

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<sup>5</sup> The quantities are conveniently computed by solving a linear program, which maximizes the total value of the allocation, subject to both the constraints that the total number of units allocated for each product must not exceed the corresponding supplies and the bidders' quantity constraints included in their individual bids. This is the dual of the pricing linear program mentioned above.

## Appendix: Assignment Auction Rules (corrected August 23)

The auction will allocate the entire quota of every available fishing license through a single and simultaneous sale. The units by which quotas are divided could be set similarly to New Zealand's auctions, with 100,000,000 shares for each license, which makes each share correspond to one-millionth of a percent of total catch in its category.

**Licenses.** A license  $j$  specifies: [i] a collection of fish species (including, for a given species, each of its accompanying fauna) and [ii] a geographical area.

**Products.** A product specifies: [i] a license  $j$  and [ii] the product type  $t$ , which is either set-aside ( $s$ ) or unrestricted ( $u$ ), that determines the category of bidders allowed to bid for and utilize that product.

For any license  $j$ , the number of shares of its set-aside product is denoted by  $s_j$  and the number of shares of its unrestricted product is denoted by  $u_j$ . Hence, the total supply of license  $j$  is  $s_j + u_j$ . For example, if the total supply of a license is 100,000,000 units with 25% set-aside, then the supplies would be  $s_j = 25,000,000$  units of the set-aside product and  $u_j = 75,000,000$  units of the unrestricted product.

Bidders express their demand for all products for which they are eligible by submitting a collection of sealed bids and associated information as described below, simultaneously for all licenses. All bids and associated information are submitted through an electronic auction system. The information must include base bids and can also optionally specify bid groups, bid weights, and group limits.

**Base Bids.** Each base bid  $b$  specifies: [i] the bidder  $i_b$ ; [ii] a license  $j_b$  and a product type  $t_b \in \{s, u\}$ ; [iii] a quantity  $q_b > 0$ , and [iv] a value (price)  $v_b > 0$ . Each bidder may submit multiple base bids for each license.

For any given license  $j$ , each bidder is classified as either a set-aside bidder or an ordinary bidder. Ordinary bidders are only allowed to bid for the unrestricted product and may not win more than  $u_j$  units of license  $j$ . Set-aside bidders only bid for the set-aside product, but their bids apply to the entire supply of license  $j$ . For the sake of clarity, we emphasize that set-aside bidders may bid for and possibly win up to  $u_j + s_j$  units of license  $j$ .

**Bid Group.** A bid group  $G$  is a collection of various base bids for different products submitted by the same bidder. Each base bid can be included directly in at most one bid group.

**Bid Weight.** A bid weight  $w_{G,b}$  is a weight assigned by the bidder to the base bid  $b$  in the bid group  $G$ .

**Group Limit.** A group limit  $l_G$  constrains the quantities assigned to all base bids in the bid group  $G$ . It specifies the maximum total weighted quantity of products that the bidder may be assigned through all base bids in the bid group.

**Auction Allocation.** An allocation of products to bidders is a collection of quantities  $X_{ij}$  that, for every license and every bidder, indicates the quantity of license  $j$  that is assigned to bidder  $i$ . In describing the allocation optimization problem, let  $x_b$  indicate the winning quantity associated with base bid  $b$  (a number between 0 and  $q_b$ ). This means that the total quantity of license  $j$  that is assigned by the auction to any bidder  $i$  across all its base bids is  $X_{ij} = \sum_{b|j_b=j, i_b=i} x_b$ .

The auction selects the winning quantities from the base bids to maximize the total bid value, as described in the formula below. The maximization is constrained so that [i] each base bid  $b$  is assigned a quantity  $x_b$  between zero and the bid quantity  $q_b$ , [ii] the total assigned quantities to bidders



for license  $j$  does not exceed the total available supply for that license ( $u_j + s_j$ ), [iii] the total assignment to ordinary bidders for each license  $j$  does not exceed the unrestricted supply  $u_j$ , and [iv] the total weighted assignment to each bidder  $i$  and bid group  $G$  does not exceed the bidder-imposed limit for that group  $l_G$ .

In symbols, the auction determines the allocation by solving the following linear program:

$$\begin{aligned}
 & \max_{x_b} \sum_b v_b \cdot x_b && \text{subject to} \\
 [i] \quad & 0 \leq x_b \leq q_b && \forall b \\
 [ii] \quad & \sum_{b|j_b=j} x_b \leq u_j + s_j && \forall j \\
 [iii] \quad & \sum_{b|j_b=j, t_b=u} x_b \leq u_j && \forall j \\
 [iv] \quad & \sum_{b \in G} w_{G,b} \cdot x_b \leq l_G && \forall G
 \end{aligned}$$

**Auction-Determined Prices.** The prices of unrestricted products are determined from the preceding linear program. Denote the shadow prices of constraints [ii] <sub>$j$</sub>  and [iii] <sub>$j$</sub>  (non-negative numbers reported by the linear programming software) by  $\lambda_j$  and  $\mu_j$ , respectively. Then, the auction-determined price of the unrestricted product of license  $j$  is  $p_j^u = \lambda_j + \mu_j$ .<sup>6</sup>

The prices of set-aside products are determined as follows. Let  $p_j^{s-max}$  be the lowest shadow price of constraint [ii] <sub>$j$</sub>  in the following linear program, which involves the set-aside bidders alone and excludes any amounts they bid to buy more than  $s_j$  units.<sup>7</sup>

$$\begin{aligned}
 & \max_{x_b} \sum_{b|t_b=s} v_b \cdot x_b && \text{subject to} \\
 [i] \quad & 0 \leq x_b \leq q_b && \forall b \\
 [ii] \quad & \sum_{b|j_b=j, t_b=s} x_b \leq s_j && \forall j \\
 [iii] \quad & \sum_{b|j_b=j, t_b=s, i_b=i} x_b \leq s_j && \forall i, j \\
 [iv] \quad & \sum_{b \in G|t_b=s} w_{G,b} \cdot x_b \leq l_G && \forall G
 \end{aligned}$$

<sup>6</sup> In the unusual case that the sum is not unique, the relevant sum can be determined by making very small increases in the right-hand sides of the two constraints. It can be shown mathematically that the prices  $p^u$  are the lowest prices at which the total demand for the unrestricted units expressed by bidders through their sealed bids – including all groups and limits – equals the supply of the unrestricted product.

<sup>7</sup> The constraints [iii] do not affect the allocation in this problem, but they must be included to compute the correct shadow prices  $p_j^{s-max}$  for cases in which some constraint [iii] <sub>$ij$</sub>  is satisfied with equality.

The auction-determined price of the set-aside product of license  $j$  is  $p_j^s = \min(\lambda_j, p_j^{s-max})$ . It follows that  $p_j^s \leq p_j^u$ .

The price for the set-aside product of license  $j$  is equal to the price of the unrestricted product of license  $j$  ( $p_j^s = p_j^u$ ) when the following two conditions are satisfied: (i) the total quantity of license  $j$  won by the set-aside bidders exceeds  $s_j$  and (ii) two or more set-aside bidders are assigned some of license  $j$ .

**Auction-Determined Payments.** For all products, set-aside and unrestricted, every winning bidder pays the same price per unit. Payments are determined as follows:

- An ordinary bidder  $i$  who wins  $X_{ij}$  units of license  $j$  pays  $p_j^u X_{ij}$  for those licenses.
- A set-aside bidder  $i$  who wins  $X_{ij}$  units of license  $j$  pays  $p_j^s$  for its first units up to  $s_j$  and  $p_j^u$  for any additional units. Thus, if  $X_{ij} \leq s_j$ , then the bidder pays  $p_j^s X_{ij}$  in total for these licenses, and if  $X_{ij} > s_j$ , then it pays  $p_j^s s_j + p_j^u (X_{ij} - s_j)$ .